

Semi-finals FFJM - March 22th, 2014

Information and rankings on <http://fsjm.ch/>

START - ALL PARTICIPANTS

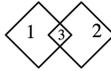
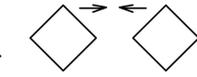
1 – MAKE A HUNDRED (Coefficient 1)

1 2 3 4 5 6 7 + + + +

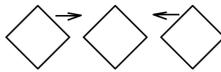
Place the four addition signs in between some of the digits from 1 to 7, without changing the order of the digits, **so as to obtain an expression whose value is equal to 100.**

2 – WITH 3 SQUARES (Coefficient 2)

By bringing the two depicted squares closer to each other horizontally, without rotating them, so that they partially overlap, one obtains a figure in which three squares are completely drawn.

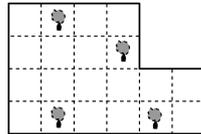


How many squares could one obtain, at most, by bringing the three squares shown closer to each other horizontally, without rotating them?



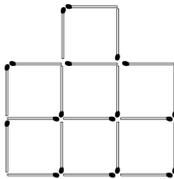
3 – ONE TREE, ONE PLOT (Coefficient 3)

The figure shows four fruit trees in a parcel of land. **Divide the land along the dotted lines into four plots**, so that the plots have identical shapes (can be superimposed), and each plot contains exactly one tree.

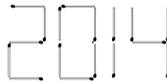


4 – THE MATCHES OF THE YEAR (Coefficient 4)

20 matches have been laid out as on the picture. After removing 3 matches, one can read the digits 2, 0, 1, and 4. Each match belongs to only one digit, and there are no matches inside the 0.

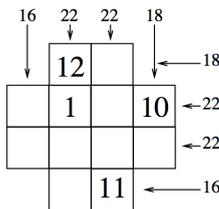


Mark the three matches that must be removed. (It is allowed to rotate the sheet of paper to read the digits. The figure shows how each digit is formed. Ignore the orientation of the individual matches.)



5 – THE CROSS OF 12 (Coefficient 5)

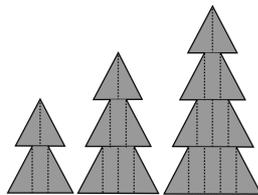
On this figure, the numbers outside the cross indicate the sum of the two or four numbers placed in the cells of the corresponding row or column. **Complete the empty cells with the numbers from 2 to 9.**



END FOR CE PARTICIPANTS

6 THE PINE TREES (Coefficient 6)

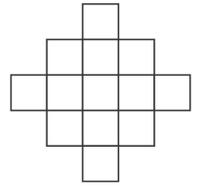
The area of the small tree at the left is 8 cm². The area of the tree in the middle is 15 cm². In these figures, all the triangles are identical and all the rectangles are identical. **What is the area of the large tree at the right in cm²?**



Note: the proportions in the figure are not accurate.

7 – ZORRO HAS ARRIVED (Coefficient 7)

Zorro has drawn his famous Z-shaped signature with the tip of his sword on this pattern representing Don Diego's cape. The Z is composed of three straight line segments, each one starting at the end point of the previous one. Each of the 13 squares of the pattern is traversed by exactly one of the line segments.



Draw the Z, given that the segments do not pass through the corners of any of the small squares.

8 – PAGE NUMBERS (Coefficient 8)

The pages of a book are numbered 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13 ... One of the digits in the number of the last page appears a total of 20 times in the numbers of all the pages of the book. If the book contained thirteen fewer pages, then that same digit would only be used 14 times in total. **How many pages are there in the book?**

END FOR CM PARTICIPANTS

Problems 9 to 18: beware! For a problem to be completely solved, you must give the number of solutions, AND give the solution if there is only one, or two solutions if there is more than one. For all problems that may admit more than one solution, there is space for two answers on the answer sheet (but there may still be a unique solution).

9 – A PROMISING NUMBER (Coefficient 9)

A strictly positive integer smaller than 2014 has the following properties:

- if one subtracts 6 from this number, the result is divisible by 6;
- if one subtracts 7 from this number, the result is divisible by 7;
- if one subtracts 8 from this number, the result is divisible by 8;
- if one subtracts 9 from this number, the result is divisible by 9.

What is the number?

10 – THE LAND OF PERE SIFLEUR (Coefficient 10)

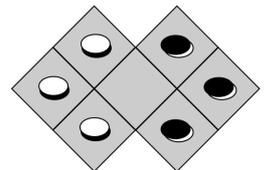
Pere Sifleur has a plot of land shaped like a right-angled trapezoid. This right-angled trapezoid can be decomposed into a rectangle and an isosceles right triangle joined together along an edge. The rectangle, whose edge lengths expressed in decameters are integers, has an area equal to 1200 m². **What is, in m², the total area of the plot of land?**

11 – EXCHANGE (Coefficient 11)

On this small board, the following moves are allowed:

- moving a stone to an adjacent square (sharing an edge), provided that square is empty;
- jumping over a stone located in an adjacent square, of any colour, provided the square beyond that stone is empty.

How many moves, at least, are necessary to exchange the positions of the white and black stones?



END FOR C1 PARTICIPANTS

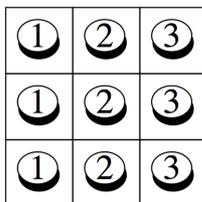
12 – FIVE SQUARES TO MAKE ONE (Coefficient 12)

Mathieu just found this wooden plank in his grandfather's attic. He wants to cut it and reassemble the pieces, laid flat, to form a large square. Moreover, he wants to do so by cutting the plank into the smallest possible number of pieces. **Draw the cuts.**



13 – THE NINE COINS (Coefficient 13)

There are nine coins: three carrying the number 1, three carrying the number 2 and three carrying the number 3, laid out as shown on a 3x3 square. A move consists of exchanging the positions of any two coins in the grid.



How many such moves are needed, at least, in order to obtain a grid in which the sums of the numbers in each of the three rows, three columns and two main diagonals are all equal to 6?

14 – AN ARITHMETIC PROGRESSION (Coefficient 14)

The first term of a sequence is $a_1 = 20$.

The second term is $a_2 = 14$.

After that, each term is given as a function of the two preceding it by the relation:

$$a_{p+2} = \frac{1 + a_{p+1}}{a_p}$$

Thus $a_3 = 3/4$, and so on.

What is the 2014th term, a_{2014} ?

If the answer is not an integer, give the answer as an irreducible fraction.

END FOR **C2** PARTICIPANTS

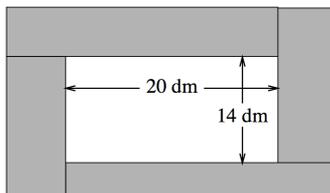
15 – DIVIDING BY 11 (Coefficient 15)

Mathilde just divided a three-digit number by 11. Surprise! There is no remainder, and the quotient is equal to the sum of the squares of the digits in the number she started with.

What was that number?

16 – MODERN ART (Coefficient 16)

The large rectangle of the figure represents a painting by famous painter Rec Tangle.



The width and height of the small white rectangle inside are respectively 20 and 14 dm.

The widths and heights of the various small grey rectangles, expressed in dm, are non-zero integers.

The four small grey rectangles have the same area.

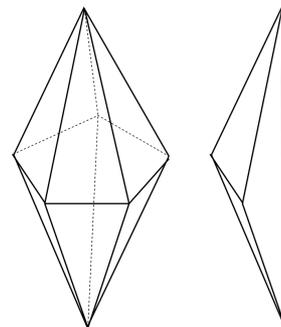
What is the smallest possible area of a small grey rectangle, expressed in dm²?

Note: the proportions of the figure are not correct. Rectangles are allowed to be squares

END FOR **L1** AND **GP** PARTICIPANTS

17 – THE BIPYRAMID (Coefficient 17)

The left figure shows a precious gem stone in the shape of a bipyramid.



There are two vertices at which 5 edges meet, and 5 vertices at which 4 edges meet. The 15 edges all have the same length, 19 mm.

One wants to cut the stone into 5 identical tetrahedra (shown at right), in which five of the six edges are edges of the bipyramid.

What is the length of the sixth edge (that which is different from the five others), expressed in mm and rounded to the nearest mm ?

If necessary, take 0.95 for $\cos(\pi/10)$.

Note: the figure does not show the correct proportions

18 – THE LUCKY YEAR (Coefficient 18)

The 1st, 2nd, 3rd, 4th terms of a sequence are respectively 13^0 , 13^1 , 13^2 , and 2014. Then each term of the sequence is equal to the sum of the four terms preceding it.

Thus the 5th term is $13^0 + 13^1 + 13^2 + 2014 = 13^3$.

How many digits are there in the 2014th term of the sequence?

If necessary, take $\log(1.928) = 0.285$ and $\log(8.748) = 0.942$, where log is base 10 logarithm.

END FOR **L2** AND **HC** PARTICIPANTS