FSJM – FINAL - May 21st 2016

Information and results at *http://fsjm.ch/*
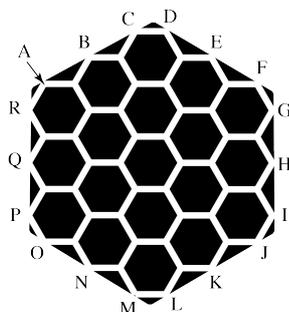
1 – Six additions (coefficient 1)

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 21    53    45    29    33    53
+31   +89   +69   +18   +51   +18
```

Mathew makes six additions. He notices that the result of one of them is exactly double that of another of them. **Which is the result which is double that of another?**

2 – The maze (coefficient 2)

Mario enters the maze via door A. He takes the first path on the left, then successively paths to the right, right, left, right, left, left, right, left, left, left, right, right, left, left, right and right and then he escapes. **By which door does he escape?**

3 – A little logic (coefficient 3)

Victor states:
- I am 14 years old;
- Fabienne is 12 years old;
- Fabienne does not always tell the truth.

Fabienne declares:
- I am 13 years old;
- Victor is also 13 years old;
- Victor does not always tell the truth.

Of the six statements by Victor and Fabienne, a maximum of how many can be true?

4 –The Coinage of Mathsland (coefficient 4)

The coinage of Mathsland is the *ludic*, composed of 100 *cents*. The only coins in circulation are the *ludic*, the 50 *cents*, the 20 *cents* and the 5 *cents*. One can pay 1.55 ludic exactly with three coins (one of one ludic, one of 50 cents, one of 5 cents), or with four coins, but it is not possible to pay exactly with 5 coins…

What is the smallest number of coins bigger than five with which it is not possible to pay 1.55 ludic exactly?

5 – The multiplication (coefficient 5)

Matilda multiplied a 4-digit number by 6, but seven of the digits in the multiplication have blown away and can be seen to the right.

But beware: a 6 and a 9 can appear identical after they have been blown away.

What was the result of Matilda's multiplication?

$$
\begin{array}{r}
\_\,0\,\_\,\_ \\
\times \quad\quad 6 \\
\hline
=\_\,\_\,0\,\_\,\_
\end{array}
$$

6 – The four squares (coefficient 6)

Write the whole numbers from 1 to 12 in the circles (the numbers 1, 9, 10, 11, 12 are already written) such that the groups of four numbers written in each of the four squares each total 24.
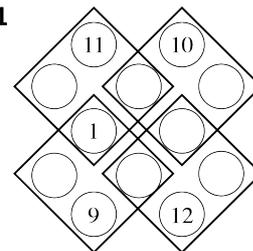
7 – The timer (coefficient 7)

On Matilda's timer each number is composed of illuminated "segments" as in the figure (six segments illuminated for a 0, two segments for a 1, five segments for a 2, etc …).
Between 00 and 59 seconds, how many times is the total number of segments illuminated equal to the sum of the two digits displayed?

8 – Adding a zero (coefficient 8)

Matilda has written down a two digit number. She forms a second number by putting a zero between the two digits of her first number, then subtracts the first number from the second. The difference is 270.

What is the first digit of her first number?

Problems 9 to 18 : beware! For a problem to be completely solved, you must give both the number of solutions, AND give the solution if there is only one, or give any two correct solutions if there are more than one. For all problems that may have more than one solution, there is space for two answers on the answer sheet (but there may still be just one solution).

9 – Semi-magic square (coefficient 9)

2	7	6	→ 15
4	3	8	→ 15
9	5	1	→ 15

18 ↘ 15 ↓ 15 ↓ 15 ↓ 6 ↙

This square is semi-magic: it is made of the whole numbers from 1 to 9 and each row and column has the same total, 15. It is not a magic square since the diagonals do not also have the total 15. Adding the totals of the two diagonals: 18 and 6, gives a result of 24.

What is the largest total one can obtain by adding the two diagonals of such a semi-magic square?

10 – Sum of the digits (coefficient 10)

Matilda writes down a 4-digit number containing three identical digits. She sums the digits of this first number to obtain a second number. She sums the digits of the second number to obtain a third number, and then sums the digits of the third number to obtain a fourth number which is 2. Her four numbers are all different.
What was Matilda's first number?

11 – The ferry (I Love Maths) (coefficient 11)

A ferry crosses from Mathsland to the Isle of Maths. After making half the distance at a constant speed, the captain, in a hurry to get home, speeds up by 25%. The ferry thus arrives half an hour before the expected time. There are no tidal or wind effects.
How long did the whole crossing take?

12 – Five numbers to find (coefficient 12)

Five (positive or negative) integers are chosen such that the ten sums that can be made from any three of them have values 3, 4, 6, 7, 9, 10, 11, 14, 15, 17.
Give the values of the smallest and largest of the five integers.

13 – Interplanetary Summit (coefficient 13)

Martians have two legs, complete with feet and toes, identical to Earthlings. But they do not have the same number of hands as Earthlings have, nor do their hands have the same number of fingers as Earthlings have.
At the first Earth-Mars summit there were six more Martians than Earthlings present. Nonetheless, the total number of fingers and toes of the Martian delegation was one fewer than their total for the Earth delegation.
How many beings in total from either planet were present in the summit?
Note: there were no amputees in the summit.

14 – Sums of two primes (coefficient 14)

Many two-digit numbers can be written as the sum of two primes. Mathew has found a two-digit number that can be equated to the sum of two primes in at least seven different ways.
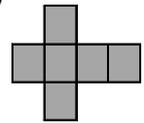**What is this number?**
Note: the primes less than 100 are: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.

15 – Pattern of the cube (coefficient 15)

Ludo wants to draw the pattern of cube, shown here, to be as large as possible on a square sheet.

Having tried to draw it with its longer axis parallel to one edge of the paper, he wonders if he could draw it larger with the longer axis along a diagonal of the paper.

From his calculation, he sees that the side of the final cube is now larger.
By what percentage, rounded to the nearest 1%?
If necessary, take 1.414 for $\sqrt{2}$

16 – Number pyramid (coefficient 16)
The positive integers are arranged as in the diagram.

0
1   **2**
3   4   **5**
6   7   8   **9**
10   11   12   13   **14**
…………………………………….

What is the sum of the first one hundred emboldened numbers?

17 – One triangle in two (coefficient 17)

An equilateral triangle is divided into two triangles such that every side in each of the two new triangles is a whole number of centimetres long.
What is the minimum length of a side of the initial triangle?

18 – Broken stick (coefficient 18)

If a straight stick is accidentally broken into three pieces, the probability of being able to form a triangle with the three pieces is 1/4.
If a straight stick is accidentally broken into four pieces, the probability of being able to form a quadrilateral with the four pieces is 1/2.
If a straight stick is accidentally broken into seven pieces,
What is the probability of being able to form a heptagon (7-sided polygon) with the seven pieces?
The answer is to be given as an irreducible fraction.
The stick is assumed perfectly straight, and broken at random independent points along its length.